Physical models for micro and nanosystems

Chapter 3: Basis of Electrostatics and Magnetostatics

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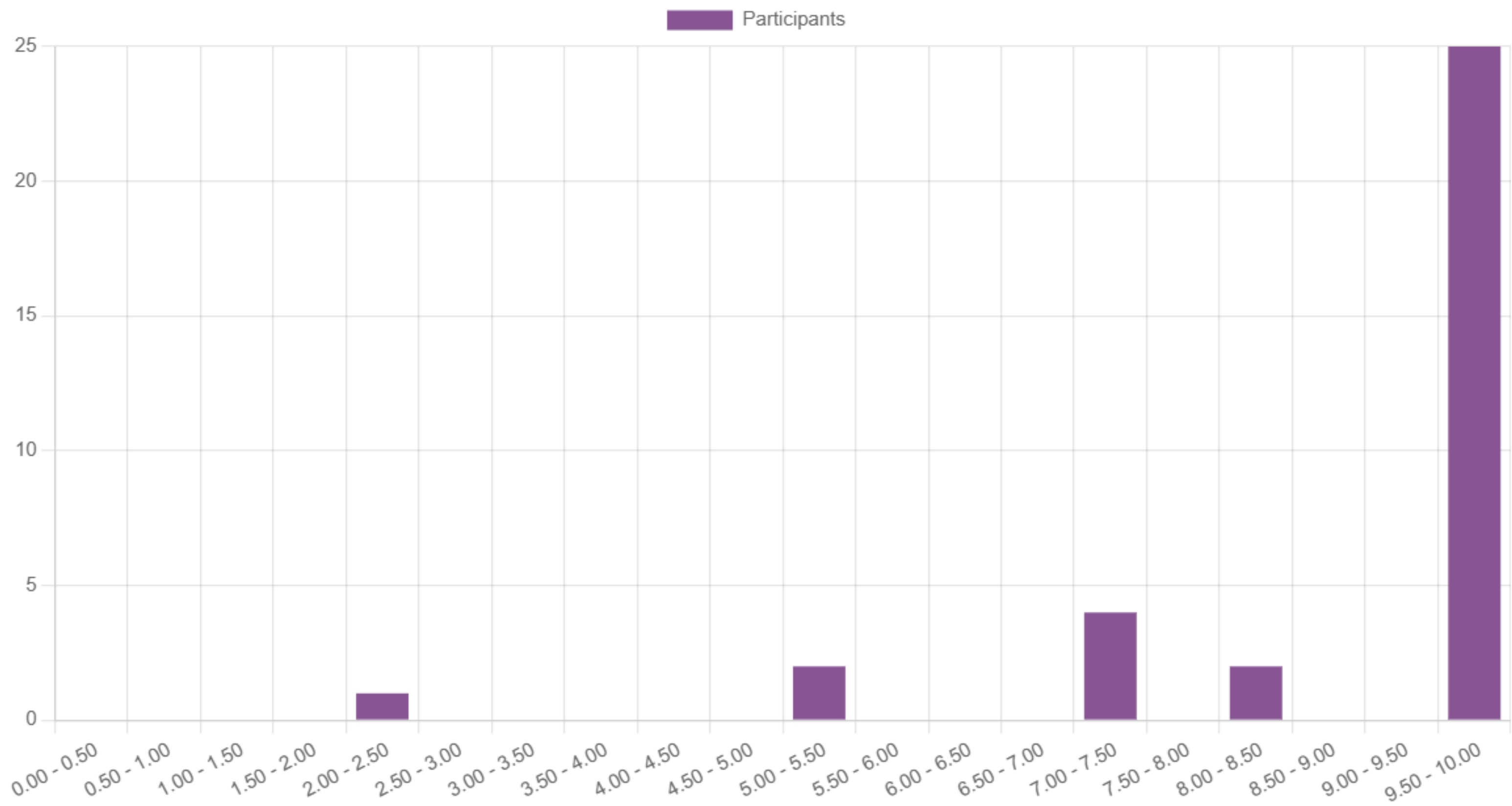
Quiz results 2024

Overall number of students achieving grade ranges



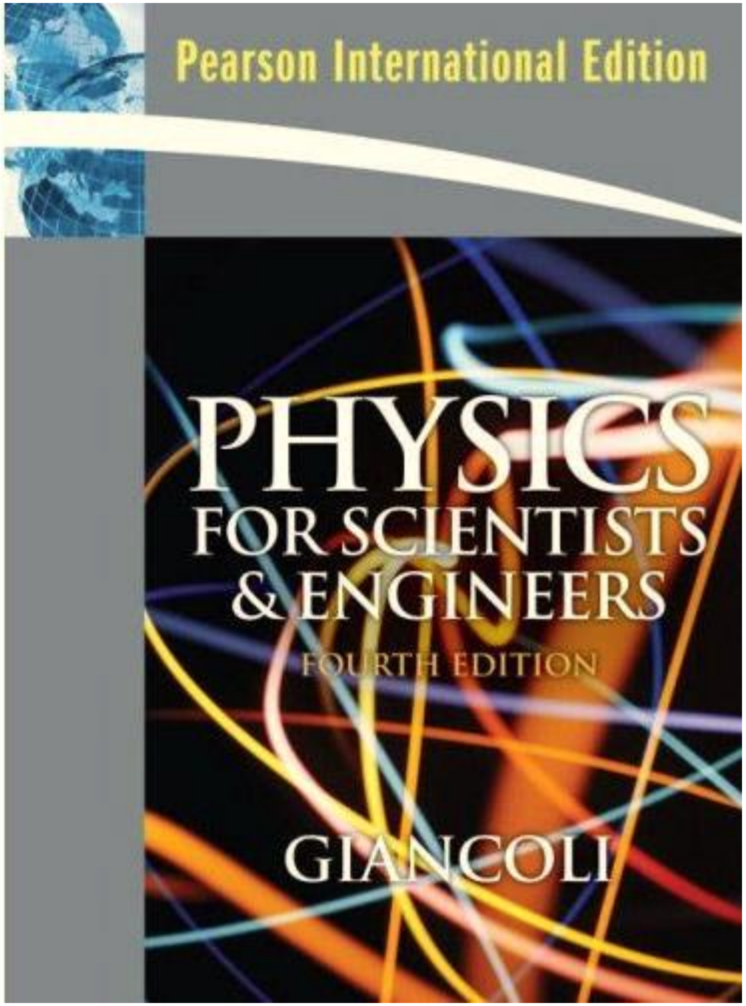
Goals

- Describe electrostatic and magnetic forces
  - Electrostatic
    - Electrostatic forces
    - Gauss law, relation to Coulomb's law
    - Capacitors
    - Electric fields in the vicinity of conductors/insulators
  - Magnetic
    - Forces acting on wires
    - Fields induced by currents
    - EMF
    - Materials
- We need to repeat this so we could perform “sanity checks” on our models
- Identify the most basic equations, preferably in scalar form

Reference

Physics for Scientists and Engineers

by Douglas C. Giancoli



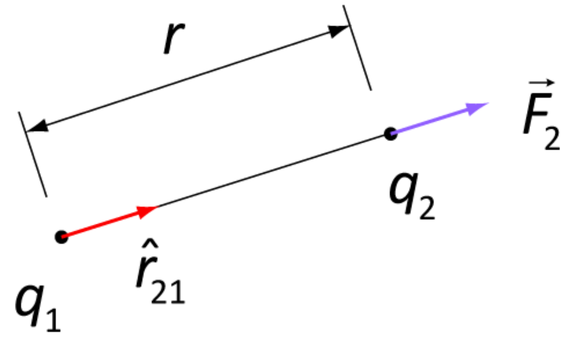
What is electrostatics?

- The word would suggest that electrostatics studies phenomena related to stationary electric charges in the absence of a magnetic field
- Absence of magnetic fields or electric currents not required
- These must however be **constant in time.**

Coulomb law

- Consider two point charges q_1 and q_2. The force acting on charge 2:

$$\vec{F}_2 = k\frac{q_1 q_2}{r^2}\hat{r}_{21}$$



$\hat{r}_{21}$vector from charge 1 to charge 2

- Constant k has the value $k = 8.99 \times 10^9\ Nm^2/C^2 \approx 9 \times 10^9\ Nm^2/C^2$
- Can also be written as

$$k = \frac{1}{4\pi\varepsilon_0},\ \varepsilon_0 = 8.85 \times 10^{-12} F/m$$ permitivity of vacuum

- Electric field – force on a positive test charge:

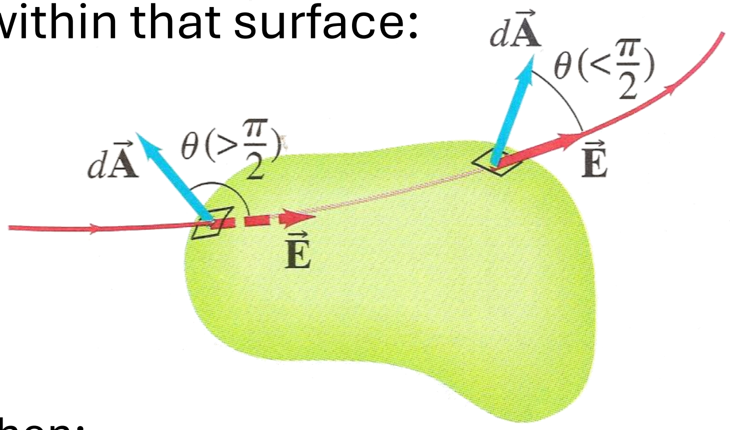
$$\vec{E} = \frac{\vec{F}}{q}$$

No need to know the charge distribution in order to predict its effect

- Superposition principle: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots$

Gauss's law

- Gives the relation between the electric field flux through a closed surface and the net charge Q_{encl} enclosed within that surface:

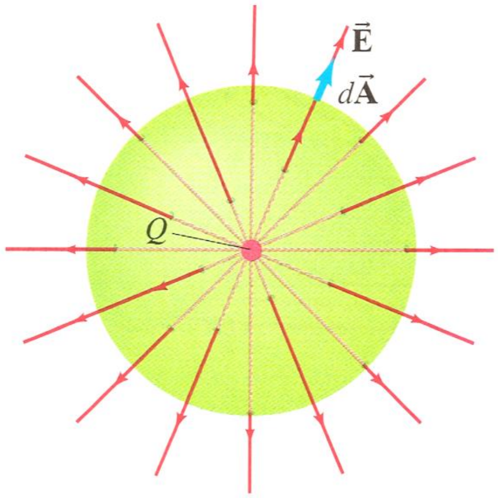


$$\Phi = \int_S \vec{E} d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

- Relation to Coulomb law

Let us imagine a spherical surface. The flux is then:

$$\int_S \vec{E} d\vec{A} = \int_S E dA = E \int dA = E \cdot 4\pi r^2$$



From Gauss's law we know that:

$$\frac{Q}{\varepsilon_0} = E(4\pi r^2)$$

Solving for E we obtain:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

Gauss's law – differential form

Differential form is obtained using the Gauss's theorem:

$$\int_S \vec{F} \cdot d\vec{A} = \int_{V(S)} \nabla \cdot \vec{F} dV$$

Gauss's law:

$$\int_S \vec{E} d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

From this we get:

$$\int_S \vec{E} d\vec{A} = \int_{V(S)} (\nabla \cdot \vec{E}) dV$$

On the right-hand side:

$$\frac{Q_{encl}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int_{V(S)} \rho dV$$

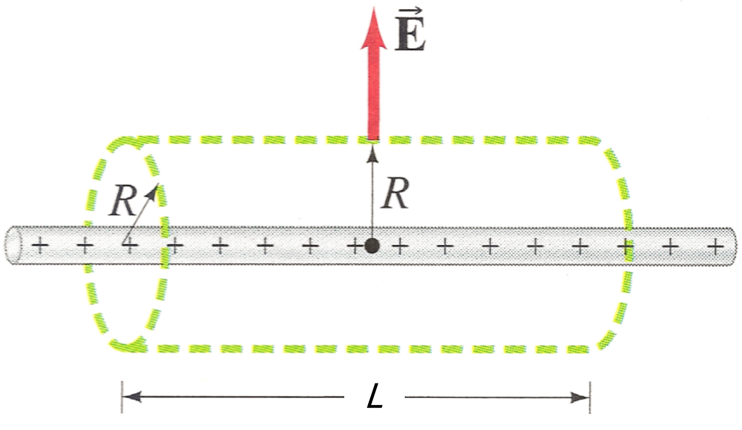
$$\int_{V(S)} (\nabla \cdot \vec{E}) dV = \frac{1}{\varepsilon_0} \int_{V(S)} \rho dV$$

$$\int_{V(S)} \underbrace{\left(\nabla \cdot \vec{E} - \frac{1}{\varepsilon_0} \rho \right)}_{=0} dV = 0 \longrightarrow \boxed{\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}}$$

Differential form of the Gauss's law

Exercise: Electric field of a line of charge

- Let us use the Gauss's law to calculate the electric field of a line of charge with linear density $\lambda = Q/L$



- For our chosen Gaussian surface, Gauss's law gives

$$\int_S \vec{E} d\vec{A} = E(2\pi RL) = \frac{Q_{encl}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

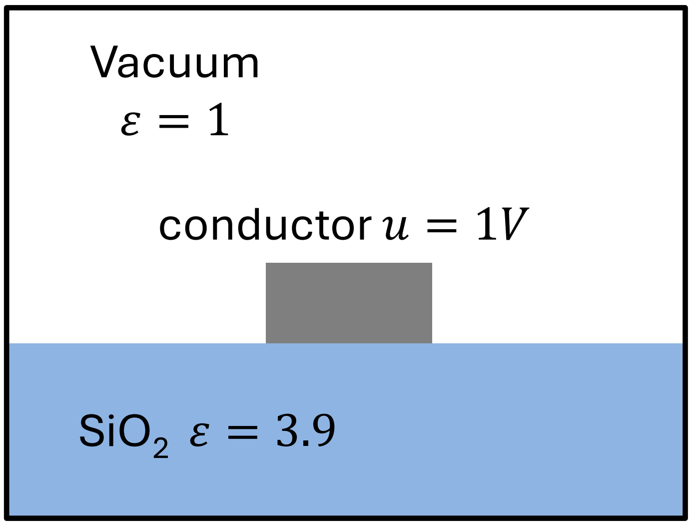
  where L is the length of our chosen surface and $2\pi R$ is its circumference. For the electric field we therefore get:

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$$

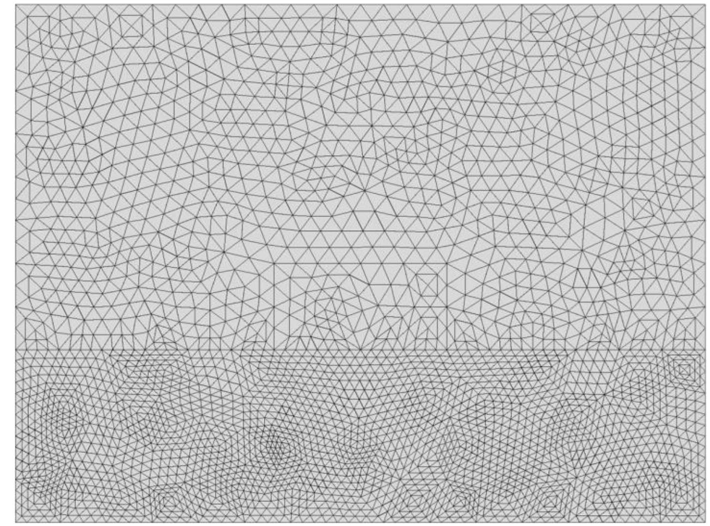
- Generalized "recipe":
  1. Recognize symettries
  2. Construct a suitable surface

Example: shielded conductor on a dielectric

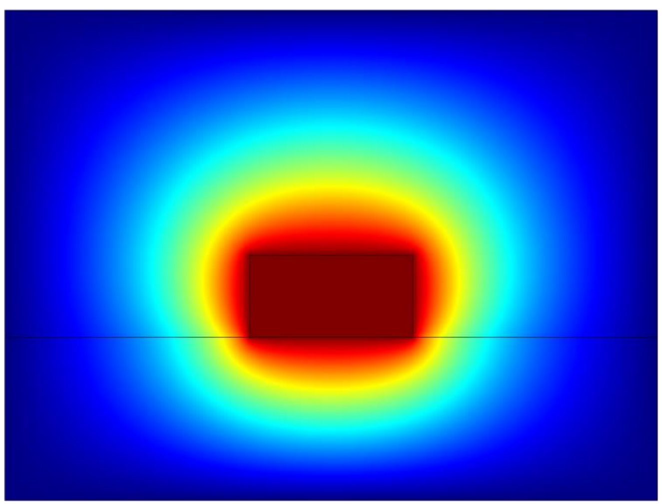
Grounded box
$u = 0V$



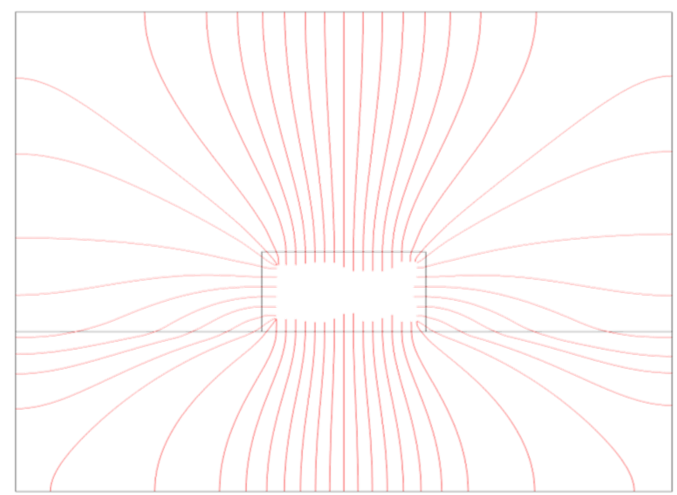
Model



Mesh



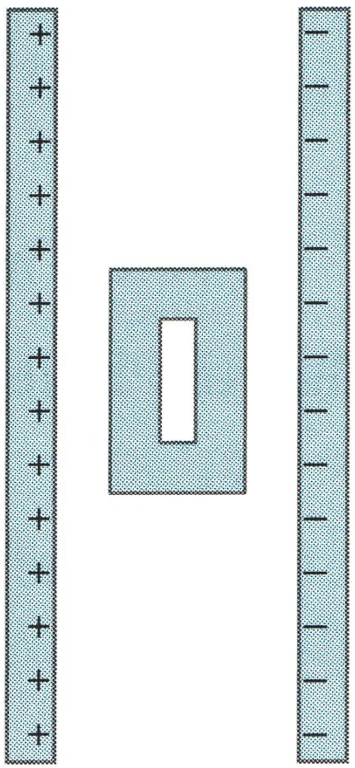
Potential distribution

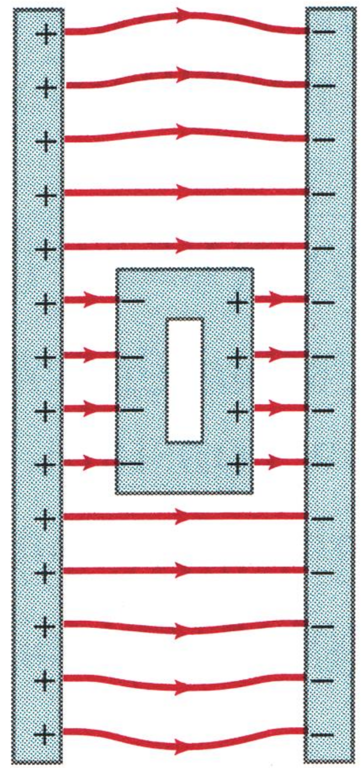


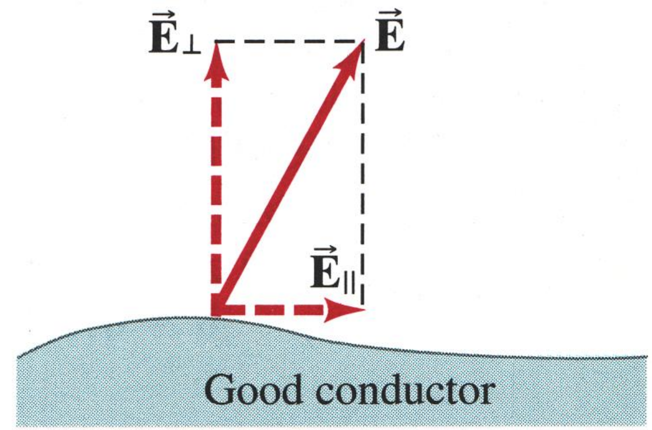
Electric field lines

Electric fields and conductors

- External electric field induces charges







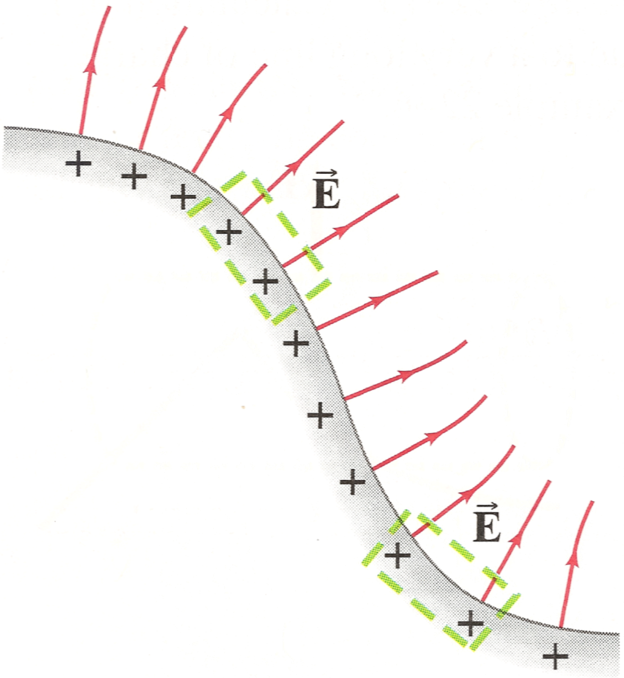
1. Electric field inside conductors is zero in a static situation
2. Close to the surface of conductors, the electric field is perpendicular to the surface

Electric field near (any) conducting surface

- We choose as our Gaussian surface a small cylindrical box so that one of its circular ends is just above the conductor. The other end is just below the conductor surface and the sides are perpendicular to it.

The electric field is:

zero inside a conductor
perpendicular just outside it
uniform over small areas



The Gauss's law then gives:

$$\int \vec{E} d\vec{a} = EA = \frac{Q_{encl}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

Where σ is the surface density of charge. We then get for the electric field:

$$E = \frac{\sigma}{\varepsilon_0}$$

Electric field of a thin charged plate (review at home)

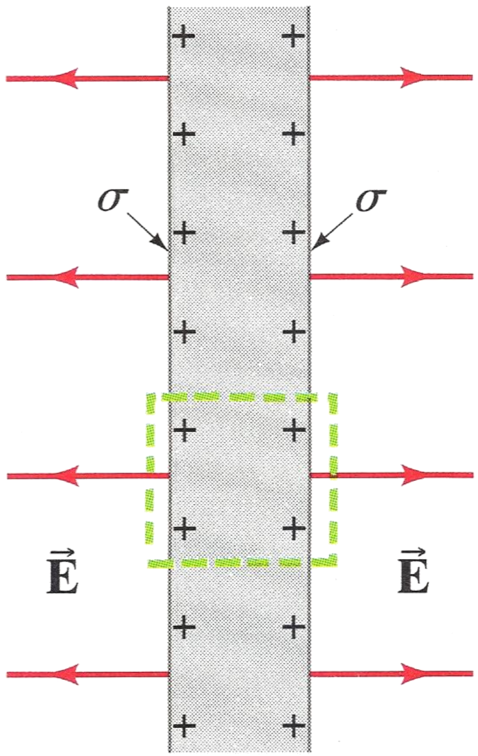
- Thin charged plate with a surface charge density σ

- For our chosen surface Gauss's law gives

$$\int_S \vec{E} d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

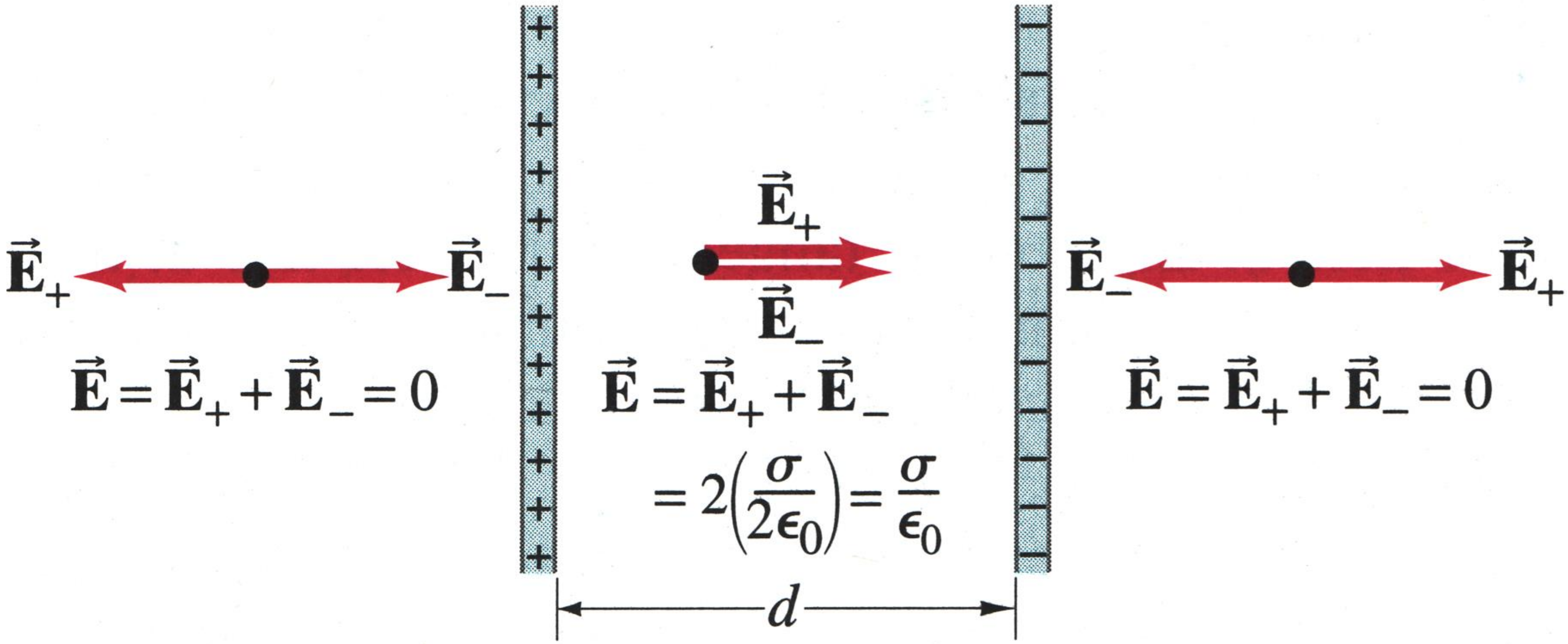
$$2EA = \frac{A\sigma}{\varepsilon_0} \longrightarrow E = \frac{\sigma}{2\varepsilon_0}$$

where A is the plate surface area



Electric field of two charged plates (review at home)

Two charged plates with a surface charge density σ



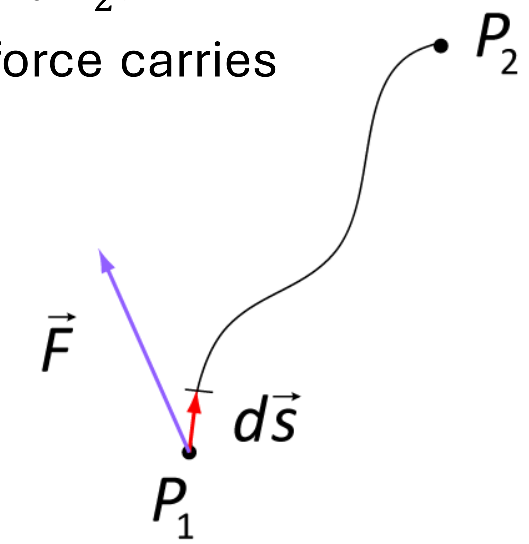
$$E = \frac{\sigma}{\varepsilon_0}$$

Does not depend on the distance!

Electric potential

Let us consider an electric field $\vec{E}$ and two points, P_1 and P_2.

In order to move a charge from P_1 to P_2 the electric force carries out work



$$W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{s} = q \int_{P_1}^{P_2} \vec{E} \cdot d\vec{s}$$

The potential difference between these two points is the potential energy difference (negative of the work done by the electric force i.e. the work you can extract) divided by the charge q:

$$U(P_2) - U(P_1) = -\frac{W}{q}$$

Electric potential

Small change of potential:

$$dU = -\vec{E} \cdot d\vec{s} \quad \text{where} \quad d\vec{s} = \hat{x}dx + \hat{y}dy + \hat{z}dz$$

and $$dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy + \frac{\partial U}{\partial z}dz$$

if we compare this with the definition of gradient from the last lecture, we can write:

$$\vec{E} = -\nabla U$$

We can then rewrite the Gauss's law as:

$$\nabla \cdot \vec{E} = \nabla \cdot (-\nabla U) = -\nabla^2 U = \frac{\rho}{\varepsilon_0}$$

$$\nabla^2 U = -\frac{\rho}{\varepsilon_0}$$ Poisson equation

Knowing that $\nabla \times (\nabla f) = 0$ for any scalar function:

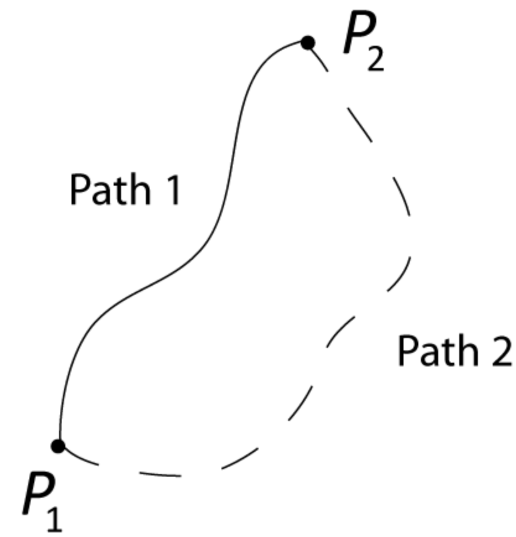
$$\nabla \times \vec{E} = 0$$

Electrostatic force is conservative (review at home)

- From the definition of electric potential:

$$W_{Path1} = \left(\int_{P_1}^{P_2} \vec{F} \cdot d\vec{s}\right)_{Path1} = -(U(P_2) - U(P_1))$$

$$W_{Path2} = \left(\int_{P_1}^{P_2} \vec{F} \cdot d\vec{s}\right)_{Path2} = -\left(U(P_2) - U(P_1)\right)$$



- From the Stokes theorem:

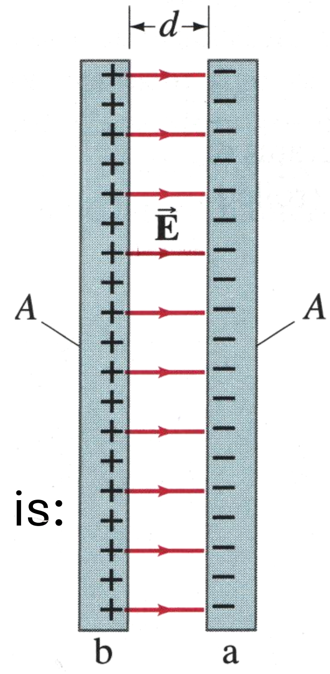
$$\Gamma_{electricfield} = \left(\int_1^2 \vec{E} d\vec{s}\right)_{Path1} + \left(\int_2^1 \vec{E} d\vec{s}\right)_{Path2} = \int_{Path1+Path2} (\nabla \times \vec{E}) d\vec{s}$$

$$\nabla \times \vec{E} = 0 \quad \rightarrow \quad \left(\int_1^2 \vec{E} d\vec{s}\right)_{Path1} + \left(\int_2^1 \vec{E} d\vec{s}\right)_{Path2} = 0 \quad \text{i.e.} \quad \left(\int_1^2 \vec{E} d\vec{s}\right)_{Path1} = \left(\int_1^2 \vec{E} d\vec{s}\right)_{Path2}$$

Electrostatic force is conservative!

Capacitance (review at home)

- Let us consider two paralel plates, spaced apart by a distance d



We assume that the distance d is small compared to the dimensions of each plate and we neglect the edge effects

The field between the plates is:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

The field is constant and uniform so that the potential ($\vec{E} = -\nabla U$) is:

$$U = U_{ba} = U_b - U_a = -\int_a^b \vec{E} \cdot d\vec{s} = \int_a^b E ds = \frac{Q}{\varepsilon_0 A}\int_a^b ds = \frac{Qd}{\varepsilon_0 A}$$

- By definition the capacitance of a capacitor is:

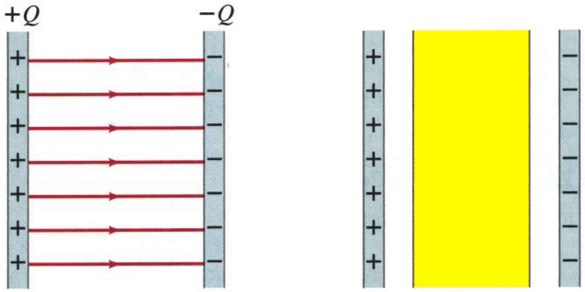
$$C = \frac{Q}{U}$$

so for the parallel plate capacitor we get:

$$C = \frac{Q}{U} = \varepsilon_0 \frac{A}{d}$$

Dielectrics

If we insert a piece of insulating material in a capacitor, the capacitance will increase by a factor κ; this is the dielectric constant



$$C = \kappa C_0 = \kappa \varepsilon_o \frac{A}{d}$$

We can define a new quantity called the permitivity of a material:

$$\varepsilon = \kappa \varepsilon_o$$

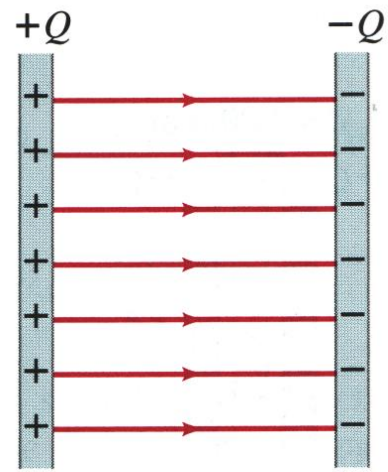
The capacitance then becomes:

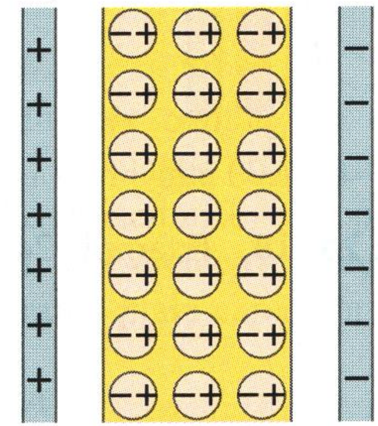
$$C = \varepsilon \frac{A}{d}$$

Material	Dielectric constant κ
Vacuum	1
Air	1.0006
Paper	3.7
SiO_2	3.9
Al_2O_3	9
HfO_2	25
Water (liquid)	80
$SrTiO_3$	300

Molecular description of dielectrics

Why should the capacitance be larger when a dielectric is present between the plates of a capacitor?

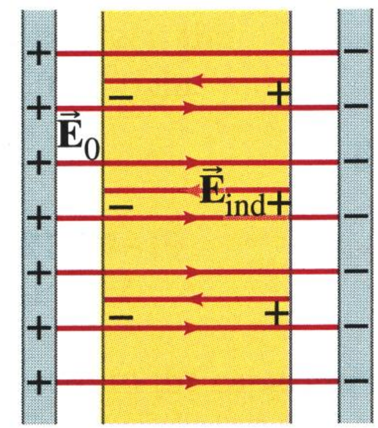




The potential difference between the plates is given by:

$$Q = C_0 U_0$$

If we insert a dielectric into the capacitor, the electric field induces surface charges



Because of this the electric field in the dielectric is smaller than in air

$\rightarrow E$ **is reduced** by a factor κ

Since $E = U/d$,

$\rightarrow$ **voltage is** also **reduced** by a factor κ

but $Q = CU$ is constant so:

$\rightarrow C$ **is increased** by a factor κ

Molecular description of dielectrics

The electric field **in the dielectric** can be considered as the vector sum of the external electric field $\overrightarrow{E_0}$ (the field in the capacitor when no dielectric is present) and the field $\vec{E}_{ind}$ due to the induced charge on the surface of the dielectric:

$$\vec{E}_D = \vec{E}_0 + \vec{E}_{ind}$$

Since these fields are acting in oposite directions, the net field is:

$$E_D = E_0 - E_{ind} = \frac{E_0}{\kappa}$$

and the induced field is:

$$E_{ind} = E_0\left(1 - \frac{1}{\kappa}\right)$$

The electric field between parallel plates is related to the surface charge on the plates

$$E_0 = \sigma_0/\varepsilon_0$$

Similarly, we can define an induced charge density:

$$E_{ind} = \frac{\sigma_{ind}}{\varepsilon_0} \quad \text{where} \quad \sigma_{ind} = \sigma_0\left(1 - \frac{1}{\kappa}\right)$$

σ_0 free charge density

σ_{ind} bound charge density

Gauss' law in dielectrics

Gauss's law:

$$\Phi = \int_S \vec{E} d\vec{a} = \frac{Q_{encl}}{\varepsilon_0}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad \text{or} \quad \nabla \cdot \left(\varepsilon_0 \vec{E}\right) = \rho_{free}$$

In dielectrics, we make the replacement

$$\varepsilon_0 \rightarrow \varepsilon = \kappa \varepsilon_0$$

So we can write:

$$\nabla \cdot \left(\varepsilon \vec{E}\right) = \rho_{free}$$

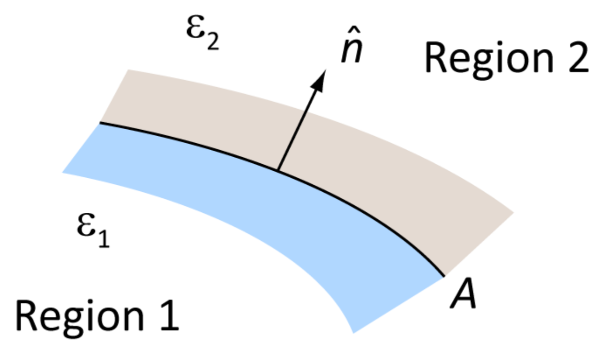
Traditionally, the magnitude $\varepsilon \vec{E}$ is referred to as the displacement field $\vec{D} = \varepsilon \vec{E}$ so Gauss's law is then written as:

$$\boxed{\nabla \cdot \vec{D} = \rho_{free}}$$

Boundary between two dielectrics (I)

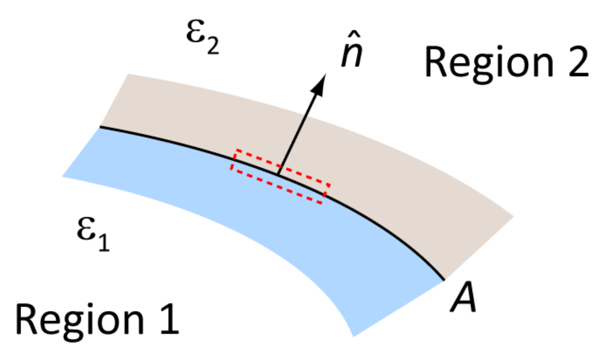
- We have two dielectric materials, 1 and 2 with respective dielectric constants κ_1 and κ_2 and free charge density σ at the interface

- We choose as our Gaussian surface a small cylindrical box so that one of its circular ends is above the interface and the other under it.



- The total flux through the surface is:

$$\Phi = \vec{D}_2 \cdot \hat{n}A - \vec{D}_1 \cdot \hat{n}A = \sigma A$$



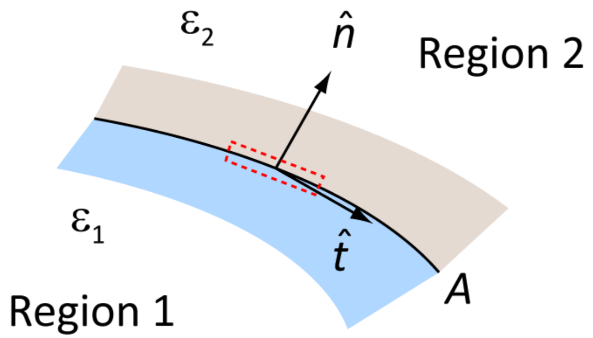
From this we get:

$$\left(\vec{D}_2 - \vec{D}_1\right) \cdot \hat{n} = \sigma$$

Boundary between two dielectrics (II)

- We can also use the Stokes theorem to get another useful boundary condition by considering a small rectangular path across the interface.

$$\nabla \times \vec{E} = 0$$



- The circulation of the electric field is:

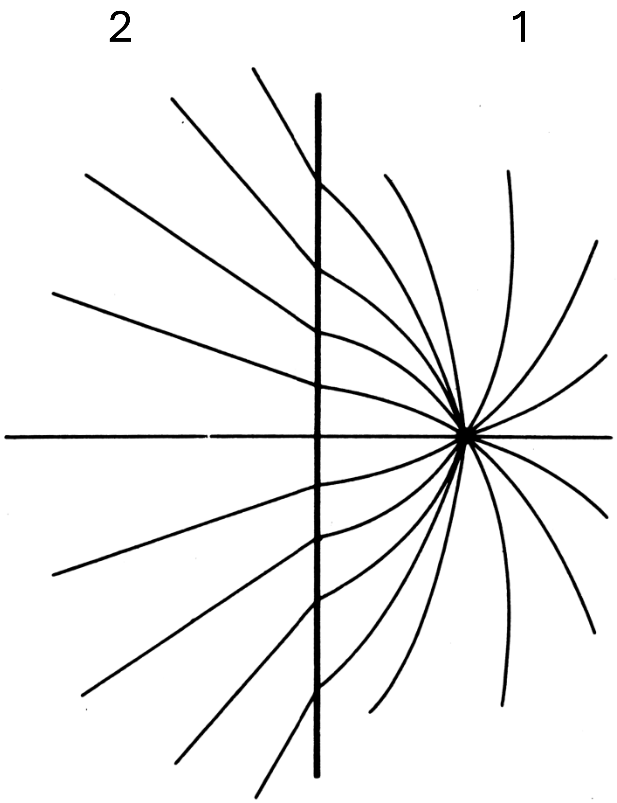
$$\Gamma = \int \vec{E} \cdot d\vec{l} = \left(\vec{E}_1 \cdot \hat{t} - \vec{E}_2 \cdot \hat{t}\right) = 0$$

The tangential component does not change:

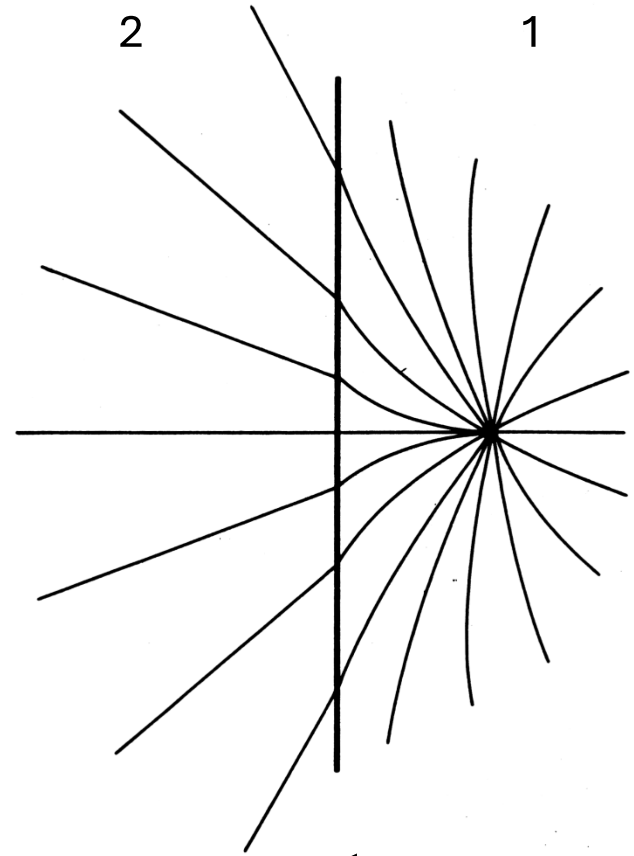
$$\left(\vec{E}_1 - \vec{E}_2\right) \cdot \hat{t} = 0$$

Boundary between two dielectrics

Electric field for a point charge close to an interface



$\varepsilon_2 > \varepsilon_1$

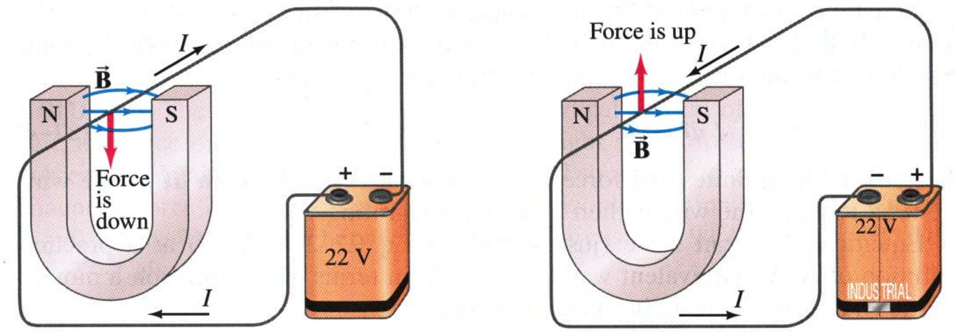


$\varepsilon_2 < \varepsilon_1$

Magnetostatics

Electric current in a magnetic field

- Magnetic fields exert force on current-carrying wires



This force is perpendicular both to the direction of the current and to the direction of the magnetic field B

- The direction is given by the right-hand rule while the magnitude is proportional to:

$$F \propto I\ell B \sin\theta$$

In the form of a vector equation:

$$\vec{F} = I\vec{\ell} \times \vec{B}$$

$\vec{\ell}$ is a vector, its magnitude is the length of the wire and direction along the wire in the direction of positive current. This formula is valid for straight wires in a uniform field. The more general form is valid for a non-uniform field and wires of arbitary shape:

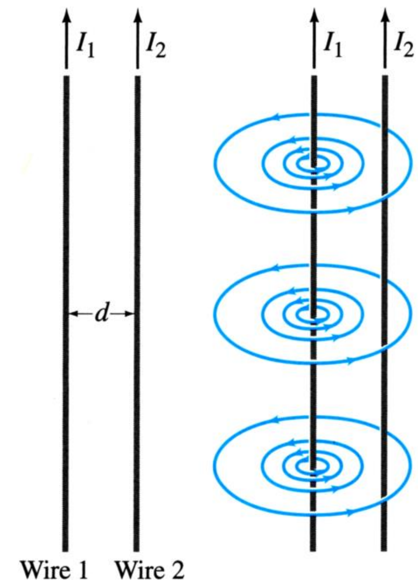
$$d\vec{F} = I d\vec{\ell} \times \vec{B}$$

Magnetic field due to a straight wire

- The magnetic field due to electric current in a long wire is:

$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$ ← We will get to this shortly

with $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Tm/A}$ (permeability of free space)



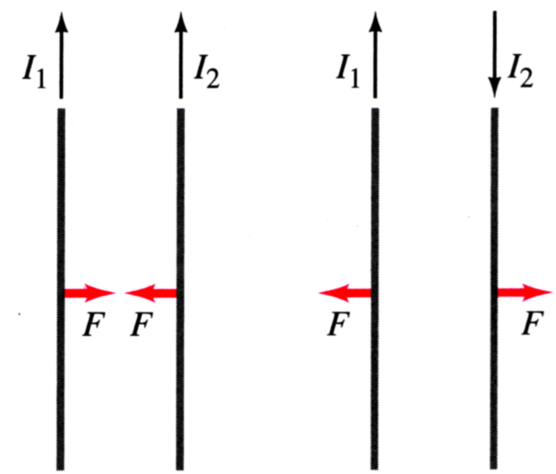
- What is the force acting between two parallel current-carrying wires?

$$B_1 = \frac{\mu_0}{2\pi} \frac{I_1}{d}$$ field from wire 1 at the position of wire 2

Force on wire 2 in the magnetic field B_1:

$$F_2 = I_2 \ell_2 B_1 = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} \ell_2$$

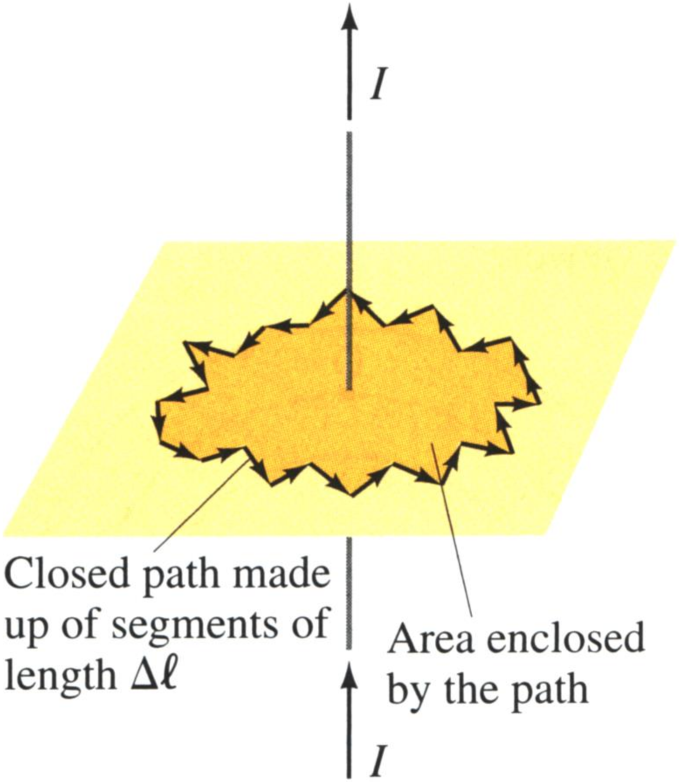
Parallel currents attract each other, while currents flowing in the opposite direction repel each other



Ampère's law

- General relation between the current in a wire (of any shape) and the magnetic field around it
- Consider an arbitrary closed path around a current. The circulation of the magnetic field around this closed path will be μ_0 times the enclosed current:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{encl}$$



- Application to the case of a straight wire:

$$\mu_0 I = \oint \vec{B} \cdot d\vec{\ell} =$$

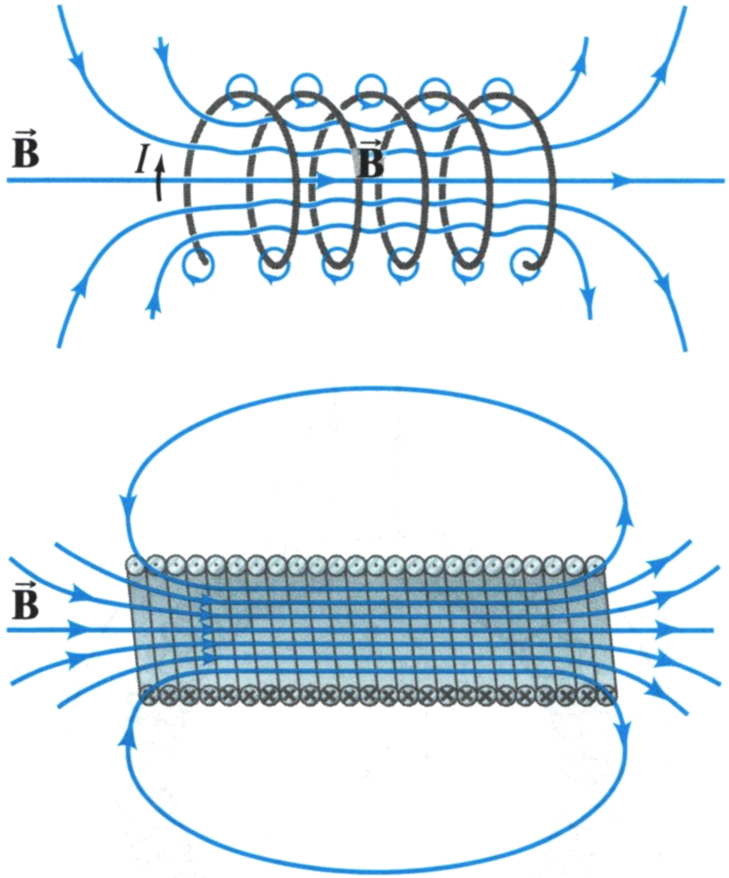
$$= \oint B d\ell = B \oint d\ell = B(2\pi r)$$

From this, the magnetic field of a straight wire is:

$$B = \frac{\mu_0 I}{2\pi r}$$

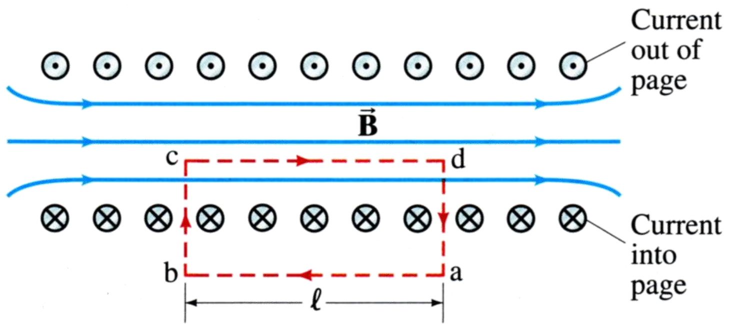
Magnetic field of a solenoid (Homework)

- Let us apply the Ampère's law to find the magnetic field of a solenoid
- The field is small outside the solenoid (except near the end) and uniform inside the solenoid



Magnetic field of a solenoid (Homework)

- Let us apply the Ampère's law to find the magnetic field of a solenoid
- The field is small outside the solenoid (except near the end) and uniform inside the solenoid
- We choose the path abcd for applying the Ampère's law. We then get:



$$\oint \vec{B}\cdot d\vec{\ell} = \int_a^b \vec{B}\cdot d\vec{\ell} + \int_b^c \vec{B}\cdot d\vec{\ell} + \int_c^d \vec{B}\cdot d\vec{\ell} + \int_d^a \vec{B}\cdot d\vec{\ell}$$

Only the integral over the cd segment is non-zero, where $\vec{B}$ is uniform and parallel to $d\vec{\ell}$ so we get:

$$\oint \vec{B}\cdot d\vec{\ell} = \int_c^d \vec{B}\cdot d\vec{\ell} = B\ell$$

The total current enclosed by the loop is NI where N is the number of loops contained in the path, so the Ampère's law gives us:

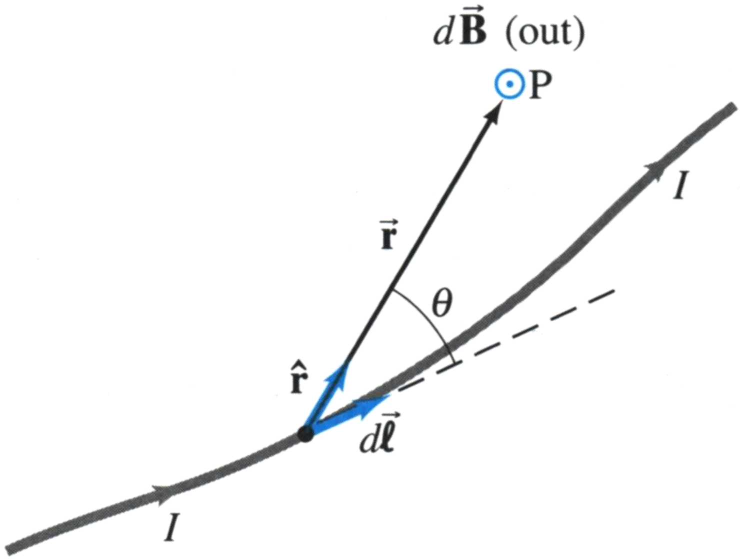
$$B\ell = \mu_0 NI$$

If we let $n = N/\ell$ be the number of loops per unit length, then:

$$B = \mu_0 nI$$

Biot-Savart Law

- Describes the magnetic field coming from a wire of any shape
- The current flowing in any path can be considered as many infinitesimal current elements. If $d\vec{\ell}$ is an infinitesimal part of the current path, the magnetic field in any point P in space is given by:



$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{r}}{r^2}$$

The total magnetic field at point P is then found by integrating over all current elements:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\ell} \times \hat{r}}{r^2}$$

Biot –Savart Law

Biot-Savart Law - example

- **Magnetic field $\boldsymbol{B}$ due to a wire segment**

One quarter of a circular loop of wire carries current I as shown in the figure. The current enters and leaves on straight segments of the wire. Find the magnetic field at point C.

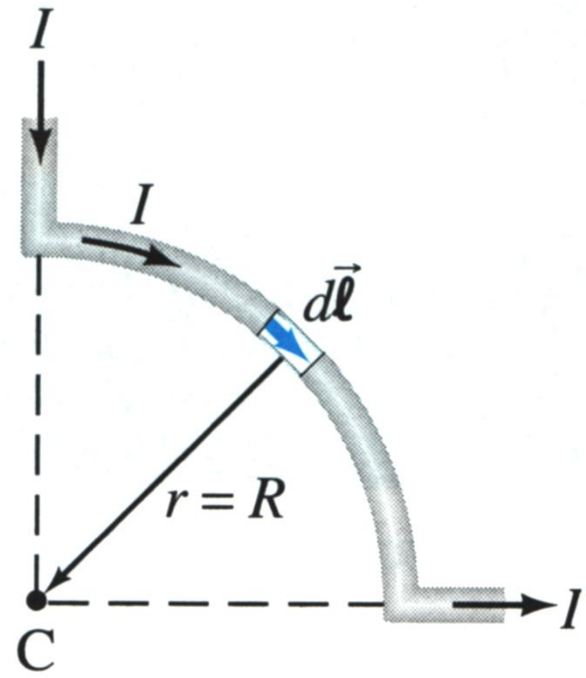
The magnetic field from straight sections is zero because $d\vec{\ell}$ and $\hat{r}$ are parallel so $d\vec{\ell} \times \hat{r} = 0$.

From the circular part we get:

$$dB = \frac{\mu_0 I}{4\pi} \frac{d\ell}{R^2}$$

We integrate over a quarter of a circle and get:

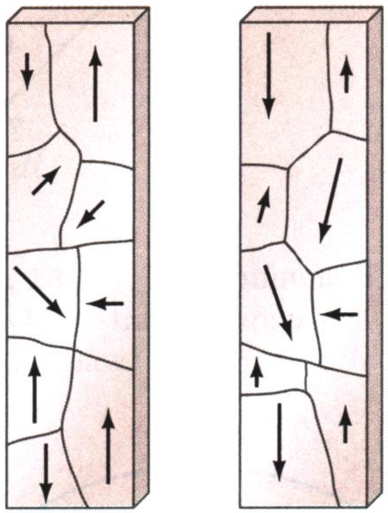
$$B = \int dB = \frac{\mu_0 I}{4\pi} \int \frac{d\ell}{R^2} = \frac{\mu_0 I}{4\pi R^2}\left[\frac{1}{4} 2\pi R\right] = \frac{\mu_0 I}{8R}$$



Magnetic field in materials: ferromagnetic

- Permanent magnets – **ferromagnetic** materials (Fe, Ni, Co, Gd only ferromagnetic elements)

- If we insert a piece of ferromagnetic material inside a solenoid the field can be increased, often by a factor of 100-1000. The resulting field is the sum of the field due to current and the field due to the material:



$\vec{B} = \vec{B}_0 + \vec{B}_M$ where $\vec{B}_M$ is the additional field due to the ferromagnetic material

The total field inside a solenoid in this case can be written by replacing the constant μ_0 by μ, characteristic of the material inside the coil:

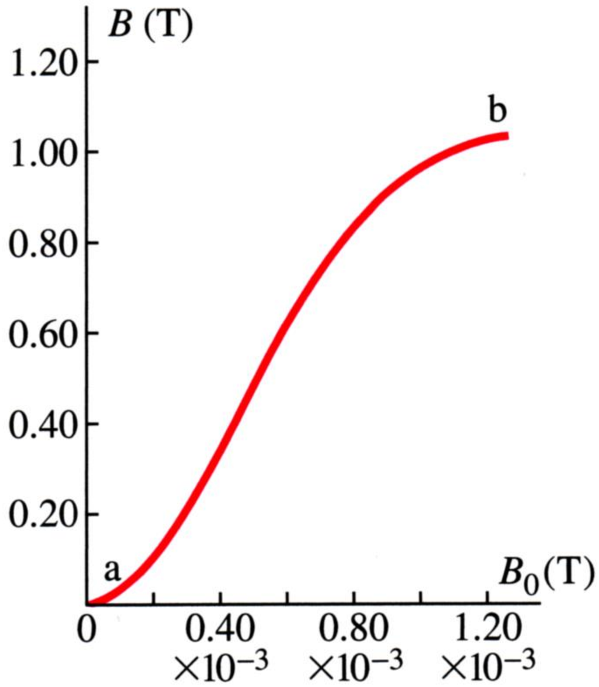
$B = \mu n I$ μ is the magnetic permeability of the material

we can introduce the field H, called the magnetic field strength and defined as:

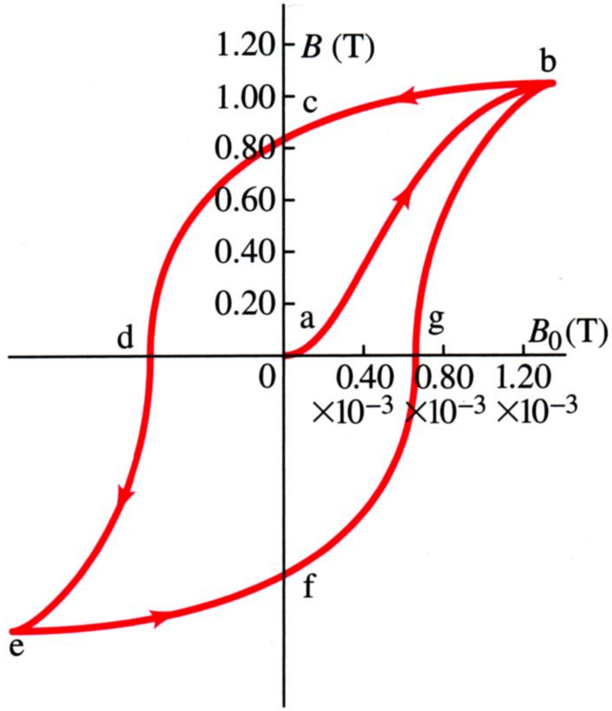
$$\vec{H} = \frac{\vec{B}}{\mu}$$

Magnetic field in materials: ferromagnetic

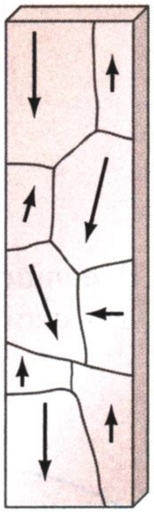
μ **is not constant**, it depends on the external magnetic field



Total magnetic field of an iron core toroid



Hysteresis curve



Magnetic field in materials

- Nonferromagnetic materials fall into two classes:
  - **paramagnetic** with $\mu > \mu_0$ – **attracted** by magnetic fields
  - **diamagnetic** with $\mu < \mu_0$ – **repulsed** by magnetic fields
- The ratio of μ to μ_0 for any material is called the **relative permeability** K_m :

$$K_\mathrm{m} = \frac{\mu}{\mu_0}$$

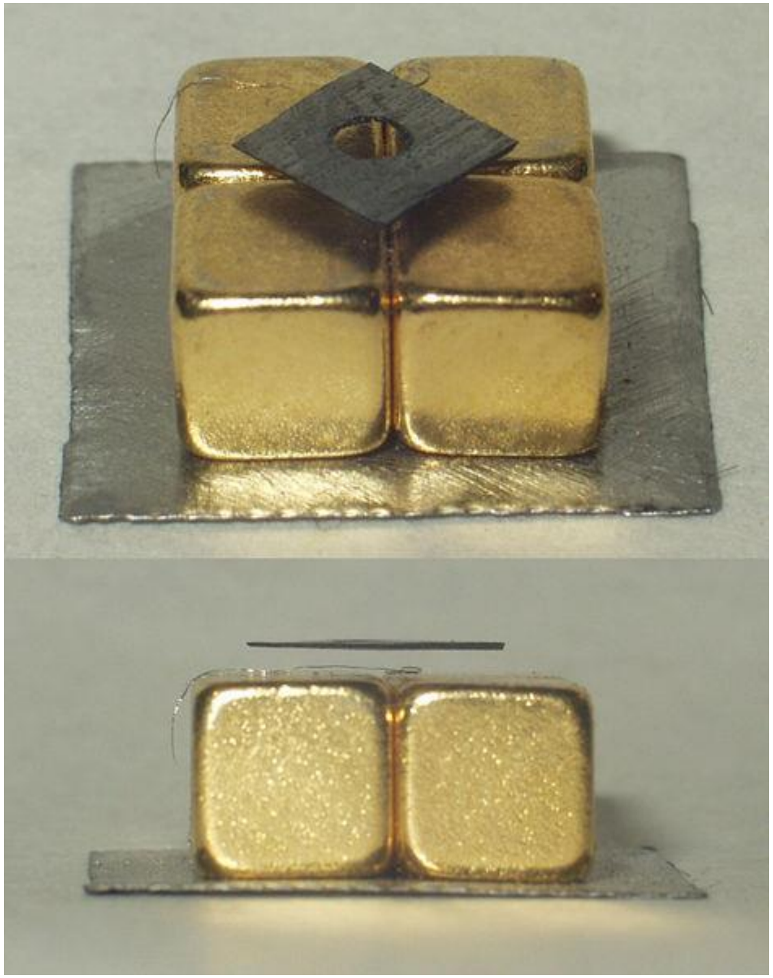
- Another useful parameter is the **magnetic susceptibility** χ_m defined as:

$$\chi_\mathrm{m} = K_\mathrm{m} - 1$$

Paramagnetic materials have $\chi_\mathrm{m} > 0$ and diamagnetic $\chi_\mathrm{m} < 0$

Paramagnetic substance	χ_m	Diamagnetic substance	χ_m
Aluminum	2.3×10^{-5}	Copper	-9.8×10^{-5}
Platinum	2.9×10^{-4}	Graphite	-1.6×10^{-5}
Tungsten	6.8×10^{-5}	Bismuth	-1.6×10^{-4}

Magnetic field in materials



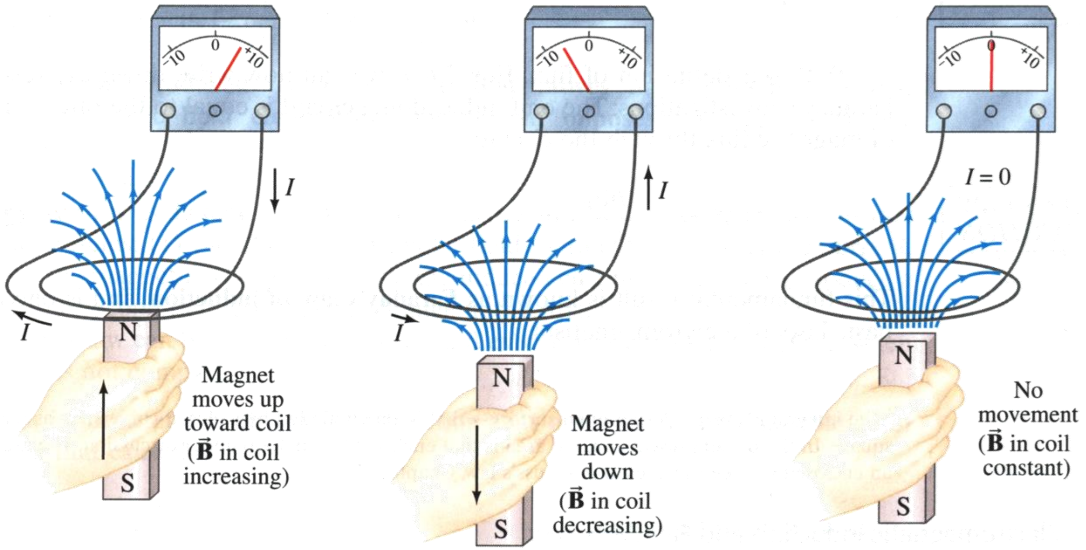
Levitating carbon

http://www.hfml.ru.nl/froglev.html

Electromagnetic induction

- Changing magnetic field induces an emf (electromotive force)

- This emf is proportional to the rate of change of the magnetic flux passing through the circuit or loop of area A. Magnetic flux is defined as:



$$\Phi_B = \int \vec{B} d\vec{A}$$

And the induced emf is:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

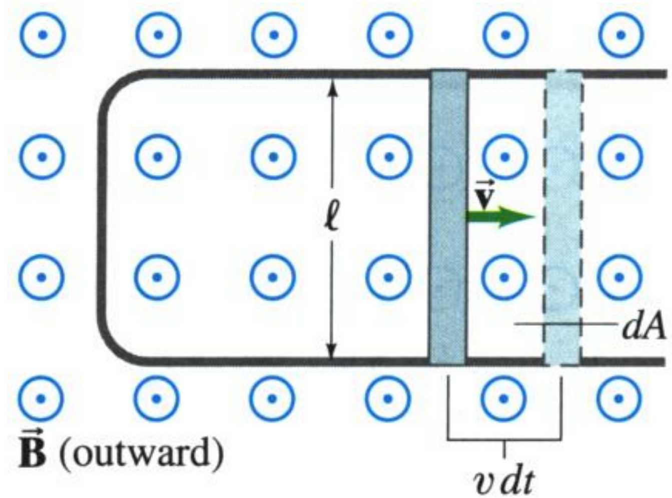
Faraday's law of induction

Lenz's law:

Current produced by an induced emf moves in such a direction that the magnetic field created by that current oposes the original change in flux.

EMF induced in a moving conductor

- Assume a uniform magnetic field B perpendicular to the area bounded by the U-shaped conductor and the movable rod resting on it
- If the rod moves at a speed v, it travels a distance $dx = vdt$ in time dt, so the area increases by an amount $dA = \ell dx = \ell vdt$
- According to Faraday's law there is an induced emf $\mathcal{E}$ whose magnitude is given by:



$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{BdA}{dt} = \frac{B\ell vdt}{dt} = B\ell v$$

Changing magnetic flux produces an electric field

A changing magnetic flux produces an electric field in any region in space (not only in conductors or wire loops), so we need to find a more general form of the Faraday's law

The potential difference between two points a and b is defined as:

$$V_{ab} = \int_a^b \vec{E} \cdot d\vec{\ell}$$

The emf $\mathcal{E}$ induced in a circuit is equal to the work done per unit charge by the electric field, which equals the integral $\vec{E} \cdot d\vec{\ell}$ along the closed path:

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell}$$

If we combine this with the Faraday's law we get:

$$\boxed{\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}}$$

General form of Faraday's law

Changing magnetic flux produces an electric field

We can rewrite Faraday's law using the Stokes's theorem:

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

On the left side we apply the Stokes theorem:

$$\oint \vec{E} \cdot d\vec{\ell} = \int_S (\nabla \times \vec{E}) \cdot d\vec{A}$$

While on the right side we can write:

$$-\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{A} = -\int_S \left(\frac{\partial}{\partial t}\vec{B}\right) \cdot d\vec{A}$$

So in the end we get:

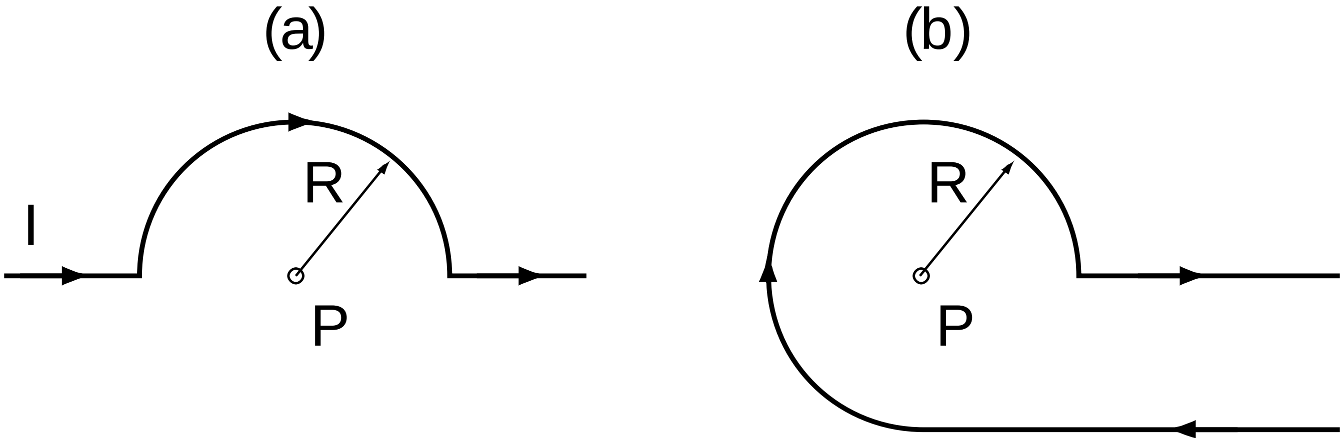
$$\int_S \left(\nabla \times \vec{E} + \frac{\partial}{\partial t}\vec{B}\right) \cdot d\vec{A} = 0 \longrightarrow \boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Differential form of Faraday's law

Nonconservative!

Exercise

Find the magnetic field at point *P* if a current-carrying wire has the shape shown in parts (a) and (b). The radius of the curved part of the wire is *R*, and the linear parts are assumed to be very long.



Homework assignement

- **Please refresh your knowledge on the subjects of:**
  - Calculating capacitance

- **Test your new/old knowledge by solving the following exercises**

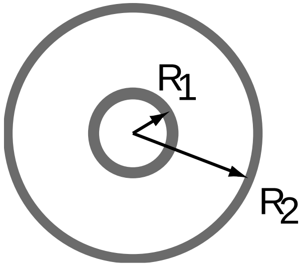
1. A thin cylindrical shell of radius R_1 is surrounded by a second concentric cylindrical shell of radius R_2. The cross-section is shown on the image. The inner shell has a total charge $+Q$ and the outer shell $-Q$. Assuming the length L of shell is much greater than R_1 or R_2 and neglecting the thickness of shells, determine the electric field as a function of the distance from center R for:

   a) $0 < R < R_1$

   b) $R_1 < R < R_2$

   c) $R > R_2$

   d) Determine the formula for capacitance



... and submit your results via moodle (exercise 2)